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SELF-FOCUSING MEDIA WITH VARIABLE INDEX OF REFRACTION

BY

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To the memory

of Rem. V. KHOKHLOV

CONTENTS

Preface

In recent years there have been many successful investigations of the optical elements, formed by inhomogeneous media. Among them are lenses and fibers; because of its wide usage for optical communication systems, the SELFOC must be specially mentioned.

Optical elements with variable refractive index have a number of peculiar characteristics that are of great interest. They can be realized in a two-dimensional form, using thin film technology. Such elements are very important in the domain of integrated optics, being widely used in systems devoted to communication, reception and processing. The development of integrated optics led to a demand for two-dimensional elements, for example modulators, deflectors, connectors, switches, filters, etc.

The present article is devoted to the problem of wave propagation in nonuniform focusing media. Besides the classical SELFOC, first described by the author in 1951, other new types of SELFOCS are considered here.

The methods used for the solution of the inverse problem of geometrical optics are also discussed. Such methods may be successfully applied for calculations of different optical elements.

It is my hope that this article will be useful not only for specialists who are interested in questions of the utilization of transparent media with given distributions of the refractive index for the realization of different types of optical elements, but also for those who are engaged in investigations of phenomena arising from the nonuniformity of the laser media in processes of generation and amplification of coherent radiation.

Lebedev Institute, Moscow, A. M. PROKHOROV
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§ 1. Introduction

1.1. STATE OF THE FIELD

In the last 15-20 years articles on the analysis of inhomogeneous media and their applications in optics have been appearing more frequently than ever, with various proposals concerned with new optical elements formed by inhomogeneous dielectrics and descriptions of the technology for the manufacture of transparent media whose refractive index varies according to a given law.

A number of specific applications of inhomogeneous media have been developed. Among them the self-focusing optical waveguides are of special interest because they are convenient for transmission of information in wide-band systems. The problem of production and application of lenses with variable refractive index has also been widely discussed. Thus a new field of optics originated, concerned with the application of inhomogeneous media to different optical elements and to systems manufacturing. This new field includes not only focusing systems, lenses and waveguides with variable refractive index, but also a number of new systems that are of interest for processing and transmitting information.

The interest in the use of inhomogeneous media has arisen not casually but is connected to a considerable degree with the development of lasers and with other advances in coherent optics. A number of possible laser applications have become of immediate importance, particularly in radio-optics, in laser engineering, in holography, in integrated optics and in various related areas.

1.2. HISTORICAL REVIEW

Maxwell was among the first to consider inhomogeneous media in optics, when, in 1854, he described a lens called "Fish-eye" (MAXWELL [1854]). This lens is a dielectric sphere whose refractive index decreases

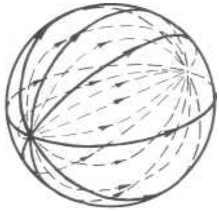

Fig. 1.1. Maxwell lens. The rays radiated from any point placed on the sphere meet at the symmetrically opposite point (MAXWELL [1854]).

from the center to the periphery according to the law

$$n(R)=\frac{n(0)}{1+(R/R_0)^2}, \tag{1.1}$$

where R_0 is the radius of the sphere, and n(0) is the refractive index in the center. This lens ensures that rays from any point located on the surface of the sphere converge to a diametrically opposite point, as shown in Fig. 1.1.

Nearly a hundred years later LUNEBURG [1944] analysed a more common type of spherical inhomogeneous medium with central symmetry where the conjugate point (focus) is situated outside the lens, as shown in Fig. 1.2. Naturally, in this case the variation of the refractive index is different and depends upon the focal lengths F, and F_2. The Luneburg lens reduces to the Maxwell lens when $F_1 = F_2 = R_o$.

In the best known Luneberg lens one of the foci is located on the sphere (F1 = R_0), and the other is located at infinity (F_2 = °°****). In this case

$$n(R)=n(0)\sqrt{2-(R/R_0)^2}, \tag{1.2}$$

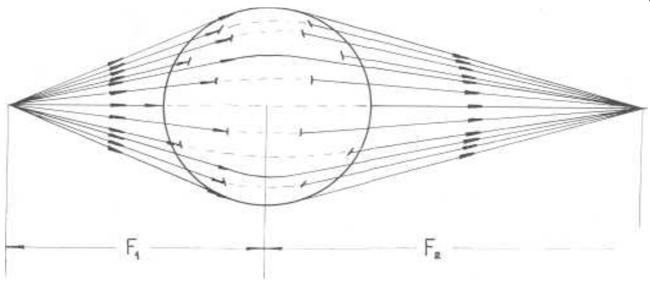

Fig. 1.2. Generalization of the Maxwell lens suggested by Luneburg; the foci are placed outside the sphere (LUNEBURG [1944]).

and by moving the source onto the lens surface, one can scan in a wide angle range without any distortion.

The properties of the Luneburg lens make it particularly attractive in connection with the microwave range scanning problem, widely discussed in the 1950's (FELD and BENENSON [1959]). Several kinds of Luneburg lenses were realized in the microwave range. The inhomogeneous medium was obtained in different ways, particularly by means of "artificial dielectric" (KOCK [1965]).

In 1951 the author described a self-focusing cylindrical medium with axial symmetry. This medium represents a dielectric waveguide where the refractive index decreases from the center to the periphery as the inverse hyperbolic cosine (MIKAELIAN [1951]):

$$n(r) = \frac{n(0)}{\operatorname{ch} \frac{1}{2}\pi r} . \tag{1.3}$$

Here r is the radius of the cylinder and $n(0)$ is the refractive index along the cylinder axis. In this case the multiple focusing of rays in propagation takes place, as shown in Fig. 1.3. This self-focusing waveguide, called SELFOC, is now widely applied as an optical fiber for wideband signal transmission, as well as for high resolution image transmission.

The cylindrical media with axial symmetry, as well as with central symmetry, serve as a variable refractive index lens. It is easily seen that a section of a self-focusing waveguide in Fig. 1.3 represents a focusing lens. This lens was studied in detail and is known as the "Mikaelian lens" in Soviet literature (FELD and BENENSON [1959], ZELKIN and PETROVA [1974], ZHOK and MOLOTSCHKOV [1973]).

The focusing properties of the medium, described by the expression (1.3), were first investigated experimentally in the microwave range (MIKAELIAN [1951]).

The laminated inhomogeneous media, described above, are the simplest ones. In 1952 the author studied more complicated cylindrical media with the refractive index depending on two coordinates (longitudinal and

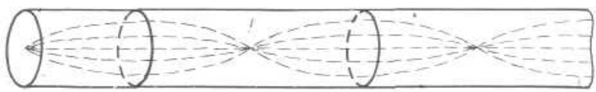

Fig. 1.3. Self-focusing cylindrical waveguide (SELFOC). If the refractive index decreases along the radius as an inverse hyperbolic cosine function, the rays radiated from the axial point source are periodically focused on the axis (MIKAELIAN [1951]).

transversal), and he showed that there exists among them an infinite number of self-focusing waveguides (MIKAELIAN [1952b]).

We will now consider some of the investigations relating to the practical realization of waveguides made of inhomogeneous dielectric. The first attempts to produce inhomogeneous optical media was made in 1964. BERREMAN [1964a,b], MARCUSE and MILLER [1964] tried to create a self-focusing waveguide as a hollow pipe, filled with inhomogeneously heated gas. In the simplest case a cold gas is driven through a heated pipe, with the gas temperature smoothly decreasing from the periphery to the center. This causes gas density variation, and, therefore, refractive index variation. Under certain conditions Marcuse and Miller succeeded in the realization of refractive index variation according to the quadratic law along the radius, a variation that is equivalent to that associated with the first and second terms of the expansion of the exact law. Because of their complexity, such "gas self-focusing waveguides" did not find practical applications.

Attempts at creating a self-focusing waveguide of fiber-glass turned out to be more successful. Such a waveguide was made by UCHIDA, FURUKAWA, KITANO, KOIZUMI and MATSUMURA [1970]. The creation of a fiberglass with variable refractive index was the decisive factor for the establishment and further development of the branch of optics that is connected with applications of homogeneous media. In recent years different types of self-focusing waveguides of practical interest have been realized. In particular they are beginning to be used as "optical cables" in communication systems. This application is of current importance, since further development of common cable communication lines is restrained by severe economic difficulties.

It is important to mention that the manufacturing process of the glassfiber with variable refractive index makes it possible to produce not only the simplest laminated self-focusing waveguides, but also waveguides with longitudinal variation of the refractive index, and some lenses of the types that we have mentioned.

It is likely that inhomogeneous media will find many useful applications in the relatively near future.

1.3. SUMMARY

This present review can be divided into two parts. In the first part laminated self-focusing media are analysed. These are the simplest inhomogeneous media, where the refractive index depends only upon a single coordinate. First we will consider lenses with central symmetry (§ 2), the Maxwell lens and the Luneburg lens being the most interesting among them. In § 3 cylindrical media with circular symmetry are considered. Among them is the optical waveguide with refractive index varying as the inverse hyperbolic cosine. In § 4 two-dimensional focusing media will be discussed.

The second part of the review is devoted to the more complicated focusing media, that are characterized by the refractive index variation. Methods for the calculation of these media are stated in § 6.

In § 7 some examples of the application are given for the calculation of cylindrical waveguides with the refractive index, that varies in the radial direction, as well as in the longitudinal direction.

It is necessary to mention that these methods, although developed a long time ago, were not used until recent times. They are of particular interest in connection with the application of inhomogeneous media in optics, and may be useful for the design and calculation of new types of light-guides, lenses and other elements with variable refractive index.

The subject under review is of interest not only for optics, but also for a number of other spheres of science and engineering, e.g., for ocean physics and geophysics, for atmospheric physics and radiophysics, etc. There is an enormous literature devoted to various phenomena involving inhomogeneous media. Many of these investigations were not concerned with optics; nevertheless, they contain useful information for our problem, especially in connection with theory, as is shown in books of ALPERT, GUINZBURG and FEINBERG [1953], TOLSTOY and CLAY [1966] and BREKHOVSKIKH [1957].

Naturally it is impossible to analyse all these studies in a single review, and a simple enumeration of them seems to be inappropriate. For this reason we will only consider in the present review some of the basic investigations that are directly concerned with applications of in-homogeneous media in optics.

In the last years I discussed certain aspects of this problem with Rem. V. Khokhlov on several occasions. Rem. V. Khokhlov was especially interested in cylindrical inhomogeneous media, whose refractive index depends on both coordinates. He studied such cases in order to gain some understanding of a number of phenomena connected with wave propagation in a laser medium, particularly in the presence of appreciable nonlinearities. Naturally, these discussions were very useful and they aided in making the present review more purposeful. I wish to mention that the idea of writing this review is to a large degree due to Khokhlov who read some of the sections (3.2, 3.3, 6.1, 6.2, § 7). On behalf of myself and his numerous friends in the field of quantum electronics this review is dedicated warmly to his memory.

§ 2. Focusing Inhomogeneous Media With Central Symmetry
§ 3. Focusing Laminated Inhomogeneous Cylindrical Medium
§ 4. Flat Laminated Inhomogeneous Focusing Media
§ 5. Experiments
§ 6. Methods for the Calculation of Inhomogeneous Focusing Media

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§ 7. Quasi-Regular Cylindrical Inhomogeneous Media

7.1. INTRODUCTION

The methods of determining inhomogeneous media with given ray paths allow us to discuss a more complicated set of optical waveguides with variable refractive index. This set of waveguides is characterized by the dependence of the refractive index not only upon the transverse coordinate, but also upon the longitudinal one. It was found (MIKAELIAN [1952a]) that there is an infinite number of SELFOCS among such waveguides, with different laws of $n(r, z)$.
As we already mentioned the problem of finding new types of SELFOCS can be formulated as an inverse problem of geometrical optics, that is, the problem of constructing inhomogeneous media with the given ray paths. In the present section this method is used for the investigation of some special types of waveguides, with the variable refractive index (MIKAELIAN [1952c, 1978]). Such waveguides are of interest not only for use as transmission lines, but for the design of new optical elements of inhomogeneous dielectrics (lenses, telescopes, focons etc.). These elements, made of waveguide sections, have an additional degree of freedom due to the refractive index variation. This degree of freedom allows us to realize the necessary wavefront transformations with lesser aberrations. This problem is an important one for the development of glass fiber systems for optical communication and for data processing systems, for integrated optics, etc.

7.2. SELF-FOCUSING WAVEGUIDES (SELFOCS)

Consider first the self-focusing waveguides where ray paths have a constant period as shown in Fig. 1.3. Since we have assumed $n(r, \varphi, z)=n(r, z)$, the pattern of ray paths will be the same at all diagonal planes, because of circular symmetry. It is therefore sufficient to investigate a two-dimensional case with ray paths being plane curves as shown in Fig. 7.1.

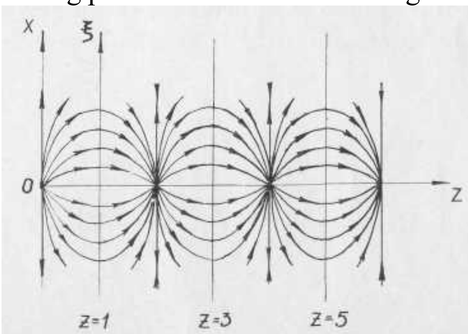

Fig. 7.1. Ray paths in a diagonal plane of a cylindrical self-focusing waveguide with the axial symmetry. The refractive index is determined by the function *n(r, z)*.

According to the given scale the ray focusing occurs on the z-axis in the points $z = 0, 2, 4, 6, \ldots$ For the planes $z = 1, 3, 5, \ldots$ the deviation of each ray reaches the maximum value ξ (for the corresponding beam). Naturally the paths must be symmetrical relative to the z-axis.
It is evident that the equation of the ray paths satisfying the above requirement may be given in different forms. This leads to a great number of cylindrical waveguides (with various *n(r, z))* where the rays are periodically focused in the course of propagation.
Consider some examples. To begin with we represent a family of curves of Fig. 7.1 in the form of an equation, that determines the ray paths for the simplest case of a laminated medium, where the refractive index does not depend on *z.* This example was investigated in § 6. In this case the refractive index is given by (MIKAELIAN [1952C]):

$$n(x, z) = \frac{\cos^2 \frac{1}{2}\pi z}{\sqrt{\mathrm{ch}^2 \frac{1}{2}\pi x - \cos^2 \frac{1}{2}\pi z}} \, \Phi\left(\frac{\mathrm{ch}\, \frac{1}{2}\pi x}{\cos^2 \frac{1}{2}\pi z}\right). \tag{7.1}$$

Hence for a cylindrical waveguide we have

$$n(r, z) = \frac{1}{\mathrm{ch}\, \frac{1}{2}\pi r} \, \Phi\left(\frac{\mathrm{ch}\, \frac{1}{2}\pi r}{\cos \frac{1}{2}\pi z}\right). \tag{7.2}$$

Here and below Φ is an arbitrary function.

The solutions (7.1) and (7.2) show that there are many self-focusing waveguides (cylindrical and plane, i.e., two-dimensional), where the rays propagate along the same paths.
It is evident that both of the expressions (7.1) and (7.2) contain the inverse hyperbolic cosine law, corresponding to the laminated waveguide.
The refractive indexes of all SELFOCS of this group vary periodically in the direction of propagation, but their period can be different. This can be easily seen on comparing the following types of self-focusing waveguides:

$$n(r,z)=\frac{1}{\mathrm{ch}\,\frac{1}{2}\pi r}\left(C_1-\frac{\cos\frac{1}{2}\pi z}{\mathrm{ch}\,\frac{1}{2}\pi r}\right), \quad (7.3)$$

$$n(r,z)=\frac{1}{\mathrm{ch}\,\frac{1}{2}\pi r}\left(C_2-\left(\frac{\cos\frac{1}{2}\pi z}{\mathrm{ch}\,\frac{1}{2}\pi r}\right)^2\right), \quad (7.4)$$

$$n(r,z)=\frac{C_3}{\mathrm{ch}\,\frac{1}{2}\pi r}\,\mathrm{th}\left(\frac{\mathrm{ch}\,\frac{1}{2}\pi r}{\cos\frac{1}{2}\pi z}\right)^2, \quad (7.5)$$

where C_1, C_2, C_3 are constants (greater than unity).
In the next example we will write the equation of the ray paths in the form

$$x(z)=\xi\cdot\sin\tfrac{1}{2}\pi z. \quad (7.6)$$

This equation also corresponds to Fig. 7.1; i.e., the focal points are located at z = 0, 2, 4, … The tangent angle of the ray output is given by

$$\mathrm{tg}\,\gamma=\mathrm{d}x/\mathrm{d}z|_{z=0}=\tfrac{1}{2}\pi\xi. \quad (7.7)$$

Using the method described in § 6, it is easily shown that for this case the self-focusing waveguides are specified by the following refractive index function:

$$r(r,z)=\mathrm{e}^{-\pi^2r^2/8}\sqrt{\left(\sin\frac{\pi}{2}z\right)^2+\left(\frac{\pi}{2}r\cdot\cos\frac{\pi}{2}z\right)^2}\;\Phi\left(\frac{\cos\frac{1}{2}z}{\mathrm{e}^{\pi^2r^2/8}}\right). \quad (7.8)$$

Varying the function Φ, one can find rather interesting variants of the refractive index laws. All of them naturally depend upon both coordinates, the dependence on the longitudinal coordinate being periodic.
Consider now the self-focusing waveguides, in which the ray paths are parallel to the z-axis in focal points z =0, 2, 4,… This means that the output angle for all of the rays is equal to zero (see Fig. 7.2).
As an example let us write the equation of the ray paths as

$$x(z)=\xi(\sin\tfrac{1}{2}\pi z)^p, \quad (7.9)$$

where "p" is a parameter, different from unity. (For an odd integer "p" the foci of all the rays coincide with the point of inflection and for even integer "p" they are the points of minima.)

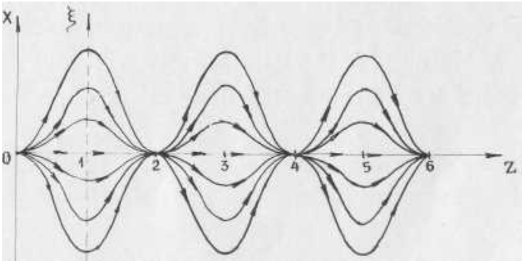

Fig. 7.2. Self-focusing waveguide with the parallel rays in a focal point.

At each focus the tangent of the outlet angle,

$$\operatorname{tg}\gamma = \left.\frac{dx}{dz}\right|_{z=0,2,\ldots} = \xi p\left(\sin\frac{\pi}{2}z\right)^{p-1}\cdot\left.\frac{\pi}{2}\cos\frac{\pi}{2}z\right|_{z=0,2,\ldots} \quad (7.10)$$

is also equal to zero. This is the reason why the excitation of such self-focusing waveguides seems to be realized only by a plane wave (more precisely, by the light source with a plane equiphase surface) in the cross-sections $z = 1, 3, 5$. As can be seen, if we move the source slightly out of the initial point (to the value of Δ_z), the output angle will be given by

$$\operatorname{tg}\gamma \cong \xi p\left(\frac{\pi}{2}\right)^{p}\Delta_z^{p-1} = \frac{\pi}{2}\,\xi p\left(\frac{\pi}{2}\Delta_z\right)^{p-1}. \quad (7.11)$$

For example,

$$\operatorname{tg}\gamma \cong \xi\frac{\pi^2}{2}\Delta_z \qquad \text{for } p = 2, \quad (7.12)$$

$$\operatorname{tg}\gamma \cong \xi\tfrac{3}{8}\pi^3\Delta_z^2 \qquad \text{for } p = 3. \quad (7.13)$$

In the case of incoherent sources (that is, without phase center) such waveguides may be more advantageous from the point of view of excitation efficiency than the convenient ones, considered above.

The laws of the refractive index distribution for this group of SELF-OCS may be found with the help of the method used in the previous case. The result is

$$n(r, z) = e^{-p\pi^2 r^2/8} \sqrt{\left(\sin\frac{\pi}{2} z\right)^2 + \left(p\frac{\pi}{2} r \cdot \cos\frac{\pi}{2} z\right)^2} \cdot \Phi\left(\frac{\cos\frac{1}{2}\pi z}{e^{p\pi^2 r^2/8}}\right). \quad (7.14)$$

It was pointed out that corresponding sections of the self-focusing waveguides represent lenses with variable refractive index. The laminated lens is a special case among the infinite variety of such lenses. For the laminated lens, as well as for the lens with the variable refractive index, where the ray paths are the same, the intensity distribution is defined by the expression (3.14).

For lenses, made of self-focusing waveguides, in which the ray paths are determined by the expression (7.6), the field distribution is approximately the same:

$$I(\xi) = \tfrac{1}{2}\pi \cos^2 \gamma = \tfrac{1}{2}\pi \frac{1}{1 + \frac{1}{4}\pi^2 \xi^2}. \quad (7.15)$$

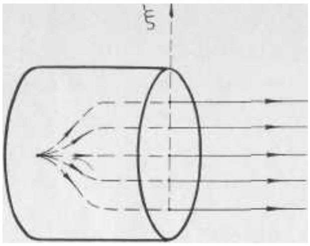

Fig. 7.3. Lens made of the inhomogeneous medium with the parallel rays in the focal point. The field distribution in the aperture can be regulated by displacement of the source.

For the third group of lenses, formed by sections of self-focusing waveguides, in which ray paths are parallel in the focal points (Fig. 7.3), the field distribution appears to be more homogeneous,

$$I(\xi) = \frac{1}{1 + p^2(\frac{1}{4}\pi^2 \Delta_z^2)^{p-1} \cdot \frac{1}{4}\pi^2 \xi^2}, \quad (7.16)$$

and can be controlled by appropriate displacement of the source from the focus.

7.3. TELESCOPIC WAVEGUIDES

Consider now waveguides with variable refractive index, where the optical length of the rays is practically the same, although the rays are not focused during propagation. An example of such a family of loci is shown in Fig. 7.4 (at a diagonal plane, as was the case earlier). One has now

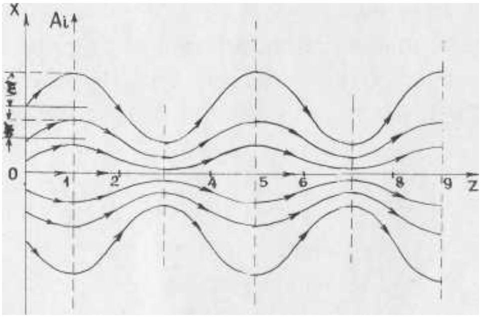

Fig. 7.4. Telescopic waveguide with the variable refractive index. The rays have the same optical length but they have no focal points (MIKAELIAN [1978]).

periodically repeated synphasic wave front. Such waveguides can be used not only as optical transmission lines, but also as telescopes with variable refractive index, which are the waveguide sections from $z = 1$ to $z=3+4k$, where k is an integer. For this reason we describe them by the term telescopic.

For example, let us represent the equation of the rays as

$$\operatorname{sh}\tfrac{1}{2}\pi x = \operatorname{sh}\tfrac{1}{2}\pi\xi(B+\sin\tfrac{1}{2}\pi z), \qquad (7.17)$$

where B is a constant. It differs from the equation (6.10), which describes the paths of the self-focusing waveguides, by an additional term which represents a shift of each ray, radiated out of the initial point, along the x-axis; the greater the parallax, the larger the output angle.

The equation of the ray family $x(z, \xi)$ can be more conveniently written as

$$\operatorname{sh}\tfrac{1}{2}\pi x = \operatorname{sh}\tfrac{1}{2}\pi A_i\left(\frac{B+\sin\tfrac{1}{2}\pi z}{B+1}\right), \qquad (7.18)$$

where, as before, each ray of the family ray is specified by the ordinate $x=A_i$ with $z = 1$.

The "magnification factor" of the simplest telescope is given by the following ratio:

$$V = \frac{x|_{z=1}}{x|_{z=3}} = A_i \Big/ \operatorname{Ar\,sh}\left(\frac{B-1}{B+1}\operatorname{sh}\frac{\pi}{2}A_i\right). \qquad (7.19)$$

With $B = 1$ the aperture is zero at the plane z=3, and the waveguide becomes a self-focusing one with the rays parallel at the focus. With B<1 the waveguide remains a telescopic one, but its ray paths have a somewhat different form (see Fig. 7.5). For this case one must take the

modulus of the ratio (7.19).
The laws of the refractive index variation of cylindric waveguides with the "telescopic" mode of ray paths (eq. (7.17)) can easily be found by applying the method, presented in section 6.3. We then find that

$$n(r, z) = \frac{1}{\cos\frac{1}{2}\pi z}\sqrt{1+B^2-\left(\frac{\cos\frac{1}{2}\pi z}{\operatorname{ch}\frac{1}{2}\pi r}\right)^2+2B\sin\frac{1}{2}\pi z}$$
$$\times\Phi\left\{\left(\frac{\cos\frac{1}{2}\pi z}{1+\sin\frac{1}{2}\pi z}\right)^B\cdot\frac{\cos\frac{1}{2}\pi z}{\operatorname{ch}\frac{1}{2}\pi r}\right\}. \qquad (7.20)$$

It is easily seen that with $B = 0$ this expression reduces to (7.2), and a telescopic waveguide becomes a self-focusing one.

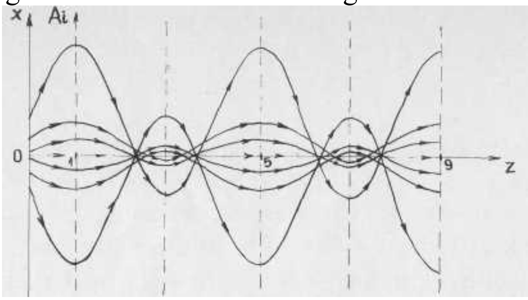

Fig. 7.5. Telescopic waveguide with the ray crossing.

For the next model of telescopic waveguides we write the ray-paths equation in the following form (in accordance with Fig. 7.4):

$$x - \xi B = \xi \sin\tfrac{1}{2}\pi z, \qquad (7.21)$$

or

$$x = \frac{A_i}{B+1}\left(B+\sin\tfrac{1}{2}\pi z\right). \qquad (7.22)$$

In this case the simplest telescope, formed by the waveguide section from $z = 1$ to z = 3, has the magnification factor

$$V = \frac{x|_{z=1}}{x|_{z=3}} = \frac{B+1}{B-1}.$$

One can show that inhomogeneous cylindrical media, corresponding to the telescopic waveguides of this group, are determined by the refractive indexes

$$n(r, z) = \sqrt{\left(\frac{B+\sin\frac{1}{2}\pi z}{\cos\frac{1}{2}\pi z}\right)^2+\left(\tfrac{1}{2}\pi r\right)^2}$$

$$\times \Phi\left\{\frac{(\cos\frac{1}{2}\pi z)^{B+1}}{(1+\sin\frac{1}{2}\pi z)^{B}} \exp\left(-\frac{\pi^2}{4}\cdot\frac{r^2}{2}\right)\right\}. \qquad (7.24)$$

With B =0 this expression reduces to (7.8), and the waveguide becomes self-focusing. With B = 1 the telescopic waveguide also becomes self-focusing, but the rays become parallel at the focus. It is easy to see that the expression (7.24) with B = 1 agrees with the expression (7.14) with p = l.
Note should be taken that the telescopes with variable refractive index have some remarkable properties. In particular, as it can be seen from (7.19) and (7.23), they may be made much more "short-focused" than the conventional ones.

7.4. IRREGULAR WAVEGUIDES WITH VARIABLE REFRACTIVE INDEX

Consider now waveguides with variable refractive index, whose properties are varied longitudinally. We will call them irregular. Strictly speaking, the waveguides described above are not regular either, with the exception of the special case of a laminated waveguide, where the refractive index varies only in the radial direction. But in these waveguides the longitudinal refractive index variation is of the periodic character. Therefore they can be considered regular on the average (during the period).
Irregular self-focusing and telescopic waveguides may be tested by the methods applied above. Here it is interesting to note two cases. In the first case the ray paths are characterized by the amplitude, varying in the course of propagation (Fig. 7.6), and for the second case the ray paths are characterized by the variable frequency (Fig. 7.7).[1]
Let us discuss a representative example: an irregular self-focusing waveguide, where the ray paths correspond to the curves plotted in Fig. 7.7a, which may be represented by the equation:

$$x(z) = \xi e^{pz} \cdot \sin\tfrac{1}{2}\pi z, \qquad (7.25)$$

where p is a parameter, characterizing the form of the ray paths. With p = 0 this equation reduces to (7.6) and corresponds to the case discussed in

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[1] Naturally other types of irregular waveguides are possible with the ray paths not quasi periodic, but represented by more complicated curves. The common methods of calculation are given in § 6.3.

§ 7.2.

It follows from eq. (7.25) that all of the rays, radiated from the initial point at different angles γ cross in the points z = 0, 2,4, 6,... In this case

$$\operatorname{tg}\gamma=\frac{\mathrm{d}x}{\mathrm{d}z}\bigg|_{z=0}=\xi e^{pz}\left(p\sin\tfrac{1}{2}\pi z+\tfrac{1}{2}\pi\cos\tfrac{1}{2}\pi z\right)\Big|_{z=0}=\tfrac{1}{2}\xi\pi, \tag{7.26}$$

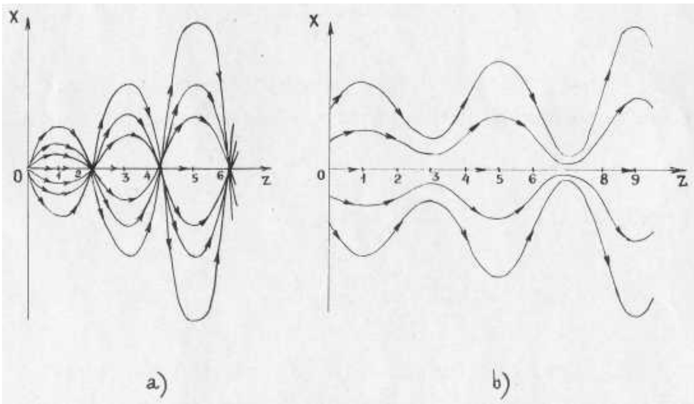

Fig. 7.6. Irregular waveguides with the variable amplitude of the rays (MIKAELIAN [1978]); (a) self-focusing waveguide; (b) telescopic waveguide.

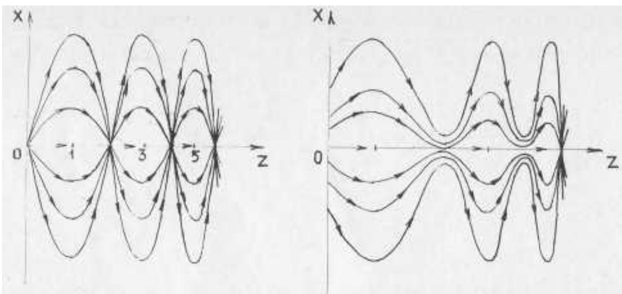

Fig. 7.7. Irregular waveguides with the crossing of the rays; (a) self-focusing waveguide; (b) telescopic waveguide.

and the equation of the ray paths can be written as:

$$x(z,\gamma)=\frac{2}{\pi}\operatorname{tg}\gamma\cdot e^{pz}\cdot\sin\tfrac{1}{2}\pi z. \tag{7.27}$$

Equiphase planes, where all the rays are parallel to the axis of the waveguide, correspond to the values of z, which are the solutions of the equation

$$p \sin \tfrac{1}{2}\pi z + \tfrac{1}{2}\pi \cos \tfrac{1}{2}\pi z = 0. \tag{7.28}$$

For the simplest particular case $p = \frac{1}{2}\pi$, the equiphase planes correspond to the values of $z = \frac{3}{2}, \frac{7}{2}, \frac{11}{2} ...$ The sections of the waveguide under consideration between the equiphase planes are telescopes. The simplest telescope "magnification factor" is equal to $e^{\pi} \approx 23$. Calculations show that this group of irregular waveguides corresponds to a dielectric cylinder where the refractive index varies according to the law:

$$n(r, z) = \sqrt{\left[\frac{\pi}{2} r\left(\sin\frac{\pi}{2} z + \cos\frac{\pi}{2} z\right)\right]^2 + \sin^2\frac{\pi}{2} z}$$
$$\times \exp\left\{-\frac{\pi}{2}\left(z + \frac{\pi}{2} r^2\right)\right\} \cdot \Phi\left(\frac{\sin\frac{1}{2}\pi z + \cos\frac{1}{2}\pi z}{e^{\pi z/2} \cdot e^{\pi^2 r^2/4}}\right). \tag{7.29}$$

From the above expressions and from Fig. 7.6 one can see that the waveguide sections from z = 0 to z = 1.5, from z = 1.5 to z = 2, from $z = 2$ to $z = 3.5$, etc., are focusing lenses, with properties different from those of self-focusing waveguides with a constant period.

With $p \neq \frac{1}{2}\pi$ the waveguide calculation is much more complicated.

The self-focusing and telescopic waveguides with variable period have remarkable advantages (Fig. 7.7). They are likely to be used as "delay systems".

SODHA, GHATAK and MALIK [1971] investigated the geometrical-optics approximation in the case where $e(r, z) = e_o - e_2(z) r^2$. They obtained solutions for several simple functions $e_2(z)$.

It is interesting to note that similar distributions of the dielectric permeability takes place in laser media, when self-focusing of the propagating pulse occurs (AKHMANOV, SUKHORUKOV and KHOKHLOV [1967]).

§ 8. Conclusion

The theory and the methods of calculation were considered for in-homogeneous media in connection with the problem of lenses and waveguides with a variable refractive index.

We have investigated some characteristic types of waveguides, discussed their properties and peculiarities; it was shown that different waveguide

sections can be used as various optical elements.
All these results may be regarded as consequences of the method of constructing inhomogeneous media for given ray paths. Naturally this method can be used in many other fields and, in particular, it solves the problem of finding new types of lenses (and other optical elements) with variable refractive index that ensure the appropriate transformations of wayefronts. It is evident that this method is applicable to cylindrical media with circular symmetry, as well as to any three-dimensional media.
In the final analysis, the success of the development of this new and important trend seems to depend on the progress of the technology of manufacturing inhomogeneous media with prescribed refractive index variation. The investigations in this domain, which began about ten years ago with the manufacture of the simplest optical elements of in-homogeneous dielectric, are, therefore, of special significance.

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